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ACCELERATED SUPERSONIC MOTION OF A PLANE AT A FINITE ANGLE OF ATTACK

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ACCELERATED SUPERSONIC MOTION OF A PLANE AT A FINITE ANGLE OF ATTACK

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ABSTRACT. Supersonic nonsteady motion of a plane in an ideal gas is considered. The plane with the attached shock wave moves at a finite (not infinitely small) angle of attack. The laws governing the change of velocity and the angle of inclination of the plane are assumed to be arbitrary, and the Struhal numbers are small. With the latter assumption, and with Mach numbers in the perturbed area not too close to unity, the motion can be characterized sufficiently well by instantaneous values of the nonsteady parameters and their derivatives. This fact provides the possibility of simplifying considerably the system of equations describing the perturbed nonsteady movement, and to obtain its analytical solution.

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A plate, forming an angle $\theta(t)$ with some fixed direction, moves in a perfect gas at supersonic velocity $U_\infty(t)$. We shall utilize the coupled system of coordinates x0y, with the x axis directed along the chord of the plate. We shall only consider flow where is an attached shock wave (Figure 1). The amplitudes of changes in U_∞ and θ can be arbitrarily large, under the condition that the shock wave is attached and that the flow behind it is supersonic. The functions $U_\infty(t)$ and $\theta(t)$ in the considered interval of time $0 \leqslant t \leqslant T$ are regarded as monotonic and continuously differentiable. The Struhal numbers and, consequently, dimensionless forward acceleration and the angular velocity are small, i.e.,

$$\begin{gathered}(\max|M_\infty^{\cdot}|, \max|\theta^{\cdot}|) \sim \varepsilon \\ (\varepsilon \ll 1) \qquad (0 \leqslant t \leqslant T) \\ M_\infty^{\cdot} = \frac{dM_\infty}{d\tau}, \quad \theta^{\cdot} = \frac{d\theta}{d\tau} \\ M_\infty = \frac{U_\infty}{a_\infty}, \quad \tau = \frac{a_\infty t}{l}\end{gathered} \tag{1}$$

*Numbers in the margin indicate pagination in the original foreign text.

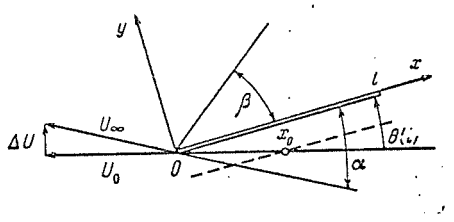

Figure 1

Here a_∞ is the speed of sound in the unperturbed medium and l is the length of the plane chord, taken as a length scale.

With such assumptions, and for Mach numbers in the perturbed area not too close to unity, the motion is characterized sufficiently well by values of the nonsteady parameters and their derivatives [1]. In that case, the unknown parameters of nonsteady relative flow of gas behind a shock wave can be represented in the form

$$c_v(s(\tau)+S),\quad \rho_\infty(\rho(\tau)+R),\quad \rho_\infty a_\infty^2(p(\tau)+P),\quad a_\infty(v+U) \tag{2}$$

where $s(\tau)$, $\rho(\tau)$, $p(\tau)$ and $v(\tau)$ are entropy, density, pressure and relative velocity of the steady stream, corresponding to the instantaneous velocity $U_\infty(t)$ and instantaneous deflection angle (Figure 1) of the stream by the plate

$$\alpha=\theta+\operatorname{arc\,tg}\frac{\Delta U}{U_0}.$$

The quantities S, R, P and U denote perturbations caused by the nonsteady state. The derivatives of s, ρ, p and v with respect to τ are of the order ε since, for instance, /71

$$\frac{\partial s}{\partial\tau}=\frac{\partial s}{\partial M_\infty}\dot{M}_\infty+\frac{\partial s}{\partial\alpha}\dot{\alpha}$$

and for the steady flow (at the Mach numbers larger than unity in the perturbed area) the derivatives of gasdynamic parameters with respect to M_∞ and α are limited.

If the values of S, R, P and U are considered to be of order ε, their derivatives with respect to spatial variables will also be of the same order; however, the derivatives with respect to τ will be of the order ε^2. If, instead

of τ, we introduce the variable $M_\infty(\tau)$, we shall obtain

$$\frac{\partial S}{\partial \tau} = \frac{\partial S}{\partial M_\infty} M_\infty^{\cdot}, \qquad \frac{\partial S}{\partial M_\infty} \sim \frac{S(\tau) - S(0)}{M_\infty(\tau) - M_\infty(0)} \sim \varepsilon$$

i.e., $\partial S / \partial \tau \sim \varepsilon^2$, and similarly for the other parameters.

Let us introduce the notation

$$q_1 = \frac{U_0}{a_\infty}, \quad q_2 = -\frac{\Delta U}{a_\infty}, \quad q_3 = \theta$$

Then for the flow with small acceleration and small angular velocity, the small kinematic parameters will be

$$q_1^{\cdot} = \frac{dq_1}{d\tau}, \qquad q_2^{\cdot} = \frac{dq_2}{d\tau}, \qquad q_3^{\cdot} = \frac{d\theta}{d\tau}$$

(We shall note that, although the theory considers coefficients of rotatory derivatives, it is more convenient here to refer the kinematic parameters to the velocity a_∞, and not to the velocity U_0 which changes in time.)

Substituting expansions of (2) into equations of the relative flow of gas, and retaining only terms of the ε order, we obtain the following system for the perturbed flow:

$$\begin{gathered} v\frac{\partial U}{\partial x} + \frac{1}{\rho}\operatorname{grad} P = \mathrm{f}(\tau), \qquad v\frac{\partial S}{\partial x} = f_4(\tau) \qquad (f = f_1 x^\circ + f_2 y^\circ) \\ v\frac{\partial R}{\partial x} + \rho \operatorname{div} U = f_3(\tau), \qquad P = a^2 R + pS \end{gathered} \tag{3}$$

Functions f, f_3 and f_4 are defined by the relations

$$\begin{aligned} f_1 &= q_1^{\cdot}\cos\theta + q_2^{\cdot}\sin\theta - v^{\cdot}, & f_3 &= -\rho^{\cdot} \\ f_2 &= -(q_1^{\cdot}\sin\theta - q_2^{\cdot}\cos\theta + 2v\theta^{\cdot}), & f_4 &= -s^{\cdot} \end{aligned}$$

Here a dot in the position of the prime denotes differentiation with respect to τ.

From (3) we have

$$(1-M^2)P_{xx}+P_{yy}=0 \qquad (M=v/a)$$

consequently,

$$P=\varphi_1(x-y\operatorname{ctg}\mu)+\varphi_2(x+y\operatorname{ctg}\mu), \qquad (\operatorname{ctg}\mu=\sqrt{M^2-1})$$

and also

$$U_x=-\frac{1}{\rho v}(\varphi_1(x-y\operatorname{ctg}\mu)+\varphi_2(x+y\operatorname{ctg}\mu))+\frac{f_1}{v}x+g_1(y) \tag{4}$$

$$U_y=-\frac{\operatorname{ctg}\mu}{\rho v}(-\varphi_1(x-y\operatorname{ctg}\mu)+\varphi_2(x+y\operatorname{ctg}\mu))+\frac{f_2}{v}x+g_2(y) \tag{5}$$

where φ_1, φ_2 and g_1 are arbitrary functions, and /72

$$g_2(y)=\frac{1}{a^2\rho}\left(-\dot{p}-a^2\rho\frac{f_1}{v}\right)y$$

The condition on the surface of the plate $U_y|_{y=0}=0$ gives

$$\varphi_1(x)=\varphi_2(x)-(\rho f_2\operatorname{tg}\mu)x \tag{6}$$

Hence

$$P=\varphi_2(x-y\operatorname{ctg}\mu)+\varphi_2(x+y\operatorname{ctg}\mu)-\rho\operatorname{tg}\mu f_2(x-y\operatorname{ctg}\mu) \tag{7}$$

$$U_y=\frac{\operatorname{ctg}\mu}{\rho v}(\varphi_2(x-y\operatorname{ctg}\mu)-\varphi_2(x+y\operatorname{ctg}\mu))+g_2(y)+\operatorname{ctg}\mu\frac{f_2}{v}y \tag{8}$$

We shall express the equation for the shock wave in the form

$$y=x\operatorname{tg}\beta(\tau)+F(x,\tau)$$

where $\beta(\tau)$ is the inclination angle of the wave for an instantaneous steady flow, and $F(x,\tau)$ is nonsteady perturbation of its form of order ε. The

relationships at the shock wave for perturbed flow are obtained by linearization of exact relations with respect to the parameter ε, and have the form

$$P = a_1 D, \quad R = a_2 D, \quad U_n = a_3 D - a_4 x, \quad U_s = a_5 \frac{\partial F}{\partial x} \tag{9}$$

Here

$$D = a_6 \frac{\partial F}{\partial x} + a_7 x, \qquad a_6 = -M_{\infty s} \cos^2 \beta, \qquad a_7 = \frac{\theta^{\cdot} + \beta^{\cdot}}{\cos \beta}$$

$$a_1 = \frac{4}{\varkappa + 1} M_{\infty n}, \quad a_2 = \frac{(\varkappa + 1) M_{\infty n}}{[1 + {}^1/_2 (\varkappa - 1) M_{\infty n}{}^2]^2}$$

$$a_3 = \frac{2}{\varkappa + 1} \left(1 + \frac{1}{M_{\infty n}{}^2}\right), \quad a_4 = \frac{\theta^{\cdot}}{\cos \beta}, \quad a_5 = -M_{\infty} \sin \alpha \cos \beta$$

Indices n and s denote projections on the external normal and tangent to the "old" wave y = x tg β, to which conditions (9) should apply under the assumptions adopted. [Conditions (9) do not contain the derivative $\partial F / \partial \tau$ for the same reason that the derivatives of the functions S, R, P and U with respect to τ are not contained in the Equations (3)].

It follows from the Expression (9) that

$$U_y = a_1 P - a_2 x \big|_{y = x \operatorname{tg} \beta}$$

$$a_1 = \frac{1}{a_1} \left(a_3 \cos \beta + \frac{a_5}{a_6} \sin \beta\right), \qquad a_2 = a_4 \cos \beta + \frac{a_5 a_7}{a_6} \sin \beta$$

From these relations, and from (7) and (8), we have the equation for $\varphi_2(x)$

$$\varphi_2(x) = \lambda \varphi_2(\delta x) + Kx$$

$$\lambda = \frac{\operatorname{ctg} \mu - \rho v a_1}{\operatorname{ctg} \mu + \rho v a_1}, \qquad \delta = \frac{1 - \operatorname{tg} \beta \operatorname{ctg} \mu}{1 + \operatorname{tg} \beta \operatorname{ctg} \mu}$$

$$K = \frac{\rho v}{(\operatorname{ctg} \mu + \rho v a_1)(1 + \operatorname{tg} \beta \operatorname{ctg} \mu)} \times$$

$$\times \left[a_2 + a_1 \rho f_2 \operatorname{tg} \mu (1 - \operatorname{tg} \beta \operatorname{ctg} \mu) + \frac{f_2}{v} \operatorname{tg} \beta \operatorname{ctg} \mu + \right.$$

$$\left. + \frac{1}{a^2 \rho} \left(a^2 f_3 + p f_4 - a^2 \rho \frac{f_1}{v}\right) \operatorname{tg} \beta \right]$$

Denoting P on the surface of the plane by $\Phi(x) = \varphi_1(x) + \varphi_2(x)$, we obtain, from the previous equation and the Expression (7), an equation for Φ /73

$$\Phi(x) = \lambda\Phi(\delta x) + (2K - (1 - \lambda\delta)\rho f_2 \operatorname{tg}\mu)x \tag{10}$$

This equation has a simple physical meaning: it characterizes the perturbation intensity of pressure, arriving at the point x* from the point δx^*, propagating along the characteristic of the first family x - y ctg μ = δx^* with subsequent reflection from the shock wave with respect to the characteristic of the second family y ctg μ + x = x*. Here λ is the reflection coefficient of perturbation from the shock wave. As shown by G. G. Cherny [2], for the case of the attached wave $|\lambda| < 1$.

The solution of the Equation (10) will be the function

$$\Phi(x) = \left(\frac{2K}{1-\lambda} - \rho f_2 \operatorname{tg}\mu\right)x \tag{11}$$

The derivatives p°, v° and β° can be easily found from the relations of an oblique discontinuity. Let the latter be expressed in the form of functions of the number M_∞ and the angle of deflection of the incoming stream α. Since

$$M_\infty = \gamma\overline{q_1^2 + q_2^2}, \qquad \alpha = \theta - \operatorname{arc\,tg}(q_2/q_1) \tag{11a}$$

we shall have

$$p^\circ = \left(\frac{\partial p}{\partial M_\infty}\frac{q_1}{M_\infty} + \frac{\partial p}{\partial \alpha}\frac{q_2}{M_\infty^2}\right)q_1^\circ + \left(\frac{\partial p}{\partial M_\infty}\frac{q_2}{M_\infty} - \frac{\partial p}{\partial \alpha}\frac{q_1}{M_\infty^2}\right)q_2^\circ + \frac{\partial p}{\partial \alpha}\theta^\circ = p^{q_1}q_1^\circ + p^{q_2}q_2^\circ + p^{q_3}q_3^\circ \tag{11b}$$

and analogous expressions for v° and β°.

As a result, (11) can be written in the form

$$\Phi(x) = (P^{q_1^*} q_1^* + P^{q_2^*} q_2^* + P^{q_3^*} q_3^*)\, x$$

$$P^{q_1^*} = \alpha_3 \left[\alpha_4 \beta^{q_1} - \alpha_5 \sin\theta + \frac{\operatorname{tg}\beta}{v} (v^{q_1} - \cos\theta) - \frac{\operatorname{tg}\beta}{a^2\rho} p^{q_1} \right] + \rho \operatorname{tg}\mu \sin\theta$$

$$P^{q_2^*} = \alpha_3 \left[\alpha_4 \beta^{q_2} + \alpha_5 \cos\theta + \frac{\operatorname{tg}\beta}{v} (v^{q_2} - \sin\theta) - \frac{\operatorname{tg}\beta}{a^2\rho} p^{q_2} \right] - \rho \operatorname{tg}\mu \cos\theta$$

$$P^{q_3^*} = \alpha_3 \left[1 + (1 + \beta^{q_3})\alpha_4 - 2\alpha_5 v + \frac{\operatorname{tg}\beta}{v} v^{q_3} - \frac{\operatorname{tg}\beta}{a^2\rho} p^{q_3} \right] + 2\rho v \operatorname{tg}\mu \qquad (12)$$

$$\alpha_3 = \frac{\rho v}{\rho v \alpha_1 + \operatorname{tg}\beta \operatorname{ctg}^2\mu}, \qquad \alpha_4 = \frac{a_5}{a_6} \operatorname{tg}\beta$$

$$\alpha_5 = \rho \alpha_1 \operatorname{tg}\mu + \frac{\operatorname{tg}\beta}{v} (\operatorname{ctg}\mu - \alpha_1 \rho v)$$

Thus, in the situation considered, the pressure perturbation on the surface is proportional to the nonsteady parameters, q_1^*, q_2^* and q_3^*. Along the plane, it is distributed according to a linear law. The remaining gasdynamic parameters can be also easily obtained.

We have to note immediately that the Expression (12) agrees with results of the linear theory of nonsteady motion of a thin-profile body. Indeed, at $\theta \to 0$, $q_2 \to 0$, $q_1 = M_\infty = M$

$$P^{q_1^*} = 0, \qquad P^{q_2^*} = \operatorname{tg}^3\mu, \qquad P^{q_3^*} = (1 - \operatorname{tg}^2\mu) / \cos\mu$$

and for the drop of pressure between the upper and the lower side of the plate /74 (in view of the antisymmetry of the stream at $\theta = 0$) we obtain

$$\left(\frac{\Delta p}{q_\infty}\right)_1 = \frac{4}{M_\infty^2} \left(-\operatorname{tg}^3\mu q_2^* - \frac{1 - \operatorname{tg}^2\mu}{\cos\mu} \theta^* \right) x, \qquad q_\infty = \frac{1}{2} \rho_\infty U_\infty^2 \qquad (13)$$

In addition, for an instantaneous steady flow

$$\left(\frac{\Delta p}{q_\infty}\right)_2 = -4 \operatorname{tg}\mu \left(\theta - \frac{q_2}{q_1} \right) \qquad (14)$$

so that the total drop $(\Delta p / q_\infty)$ is equal to the sum of Expressions (13) and (14).

If we consider only the rotation of the plate around the axis x_0 (Figure 1), then $q_2 = x_0\theta^{\cdot}$

$$\left(\frac{\Delta p}{q_\infty}\right) = -4\operatorname{tg}\mu\left[\theta + \frac{1}{M_\infty}(x - x_0)\theta - \frac{1}{M_\infty}\operatorname{tg}^2\mu\theta^{\cdot}x\right] \tag{15}$$

(Here the term containing $q_2^{\cdot}$ is neglected) as a higher order of smallness. On the other hand, according to the linear theory for a profile of zero thickness oscillating according to the law

$$y = (x - x_0)\theta, \quad \theta = \alpha e^{ikM_\infty\tau}, \quad k = \frac{\omega l}{U_\infty}$$

at small frequencies k, the pressure drop is equal to (compare [3], § 5.3)

$$q_\infty^{-1}\Delta p = -4\operatorname{tg}\mu[1 + ik(x - x_0) - ik\operatorname{tg}^2\mu x]\theta \tag{16}$$

If the motion is not harmonic, but changes slowly, then the operator ik in the Expression (16) can be replaced by $M_\infty^{-1}\partial/\partial\tau$ (the known hypothesis of harmonicity) and, as a result, we shall arrive at the Expression (15).

The middle term in this expression determines the quasi-stationary damping, and the last one (an addition due to the nonstationary state) gives the negative damping [3], if $x_0 < 2/3$.

Figures 2 - 4 show the dependence (12) of the coefficients $P_{q_1}^{\cdot}$, $P_{q_2}^{\cdot}$, $P_{q_3}^{\cdot}$ on M_∞ and α, at $q_2 = 0$.

In the calculated range of values of M_∞ and α, the value of $P_{q_1}^{\cdot}$ is positive, i.e., the longitudinal acceleration ($q_1^{\cdot} > 0$) leads to an increase in pressure in comparison with pressure corresponding to the instantaneous steady flow.

The quantities $P_{q_2}^{\cdot}$ and $P_{q_3}^{\cdot}$, in a large region of the numbers M_∞ and angles /75
α, are negative so that, in this case, the effect of a nonstationary state

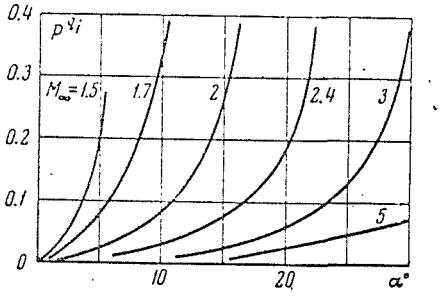

Figure 2

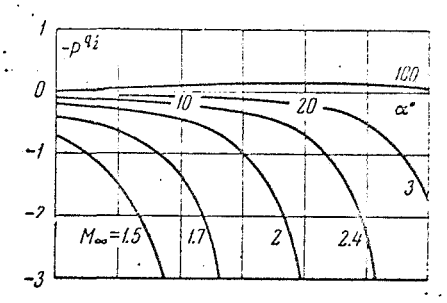

Figure 3

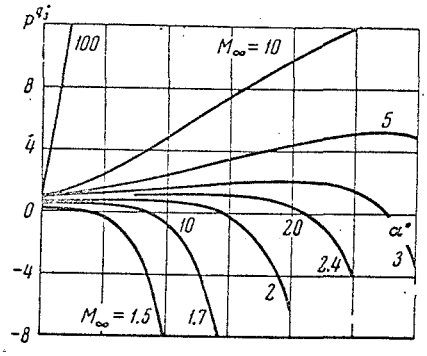

Figure 4

leads to a lowering of pressure at $q_2^\bullet < 0$ and $\theta^\bullet > 0$. A sharp increase of the absolute values of $p^{q_1^\bullet}$, $p^{q_2^\bullet}$ and $p^{q_3^\bullet}$ computed according to (12), and shown in Figures 2 - 4, takes place only when the Mach number in the perturbed area approaches unity.

Utilizing the obtained solution (11), we can determine aerodynamic characterisitcs of wedges with the semi-aperture angle θ_s, in which the attached shock wave is formed on both sides. Such calculations show that the increment of the coefficient of resistance arising due to the longitudinal acceleration

$$\Delta c_x = \frac{1}{M_\infty^2} p^{q_1^\bullet} q_1^\bullet$$

at values of $q_1^\bullet$ typical for the contempory aircraft, becomes significant only in the case where the local Mach number on the surface of the wedge approaches unity. This fact can be considered as a generalization of the known Frankel hypothesis that, for slender bodies, the longitudinal acceleration affects the resistance only at transonic velocities.

Figure 5 shows, as an example, values of the coefficients of damping moments of wedges when they rotate relative to the apex at the moment when

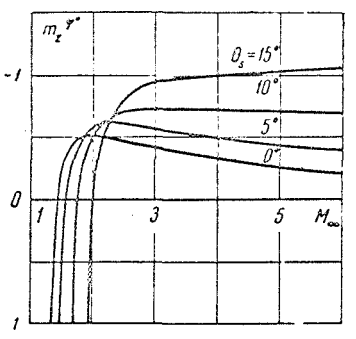

Figure 5

the angle of inclination $\varphi = 0$ ($m_z^{\varphi'}$ is related to the velocity head of the surface in a plane, the length of wedge L, and $c = LU_\infty^{-1} d\varphi / dt$). For $\theta_s = 0$, $m_z^{\varphi'}$ is determined according to the linear theory [3]. At small supersonic velocities, all wedges experience negative damping (positive values of $m_z^{\varphi'}$), which rapidly increases as the Mach number behind the shock wave approaches unity. The number M*, at which the coefficient $m_z^{\varphi'}$ changes sign, increases with the increase of the angle θ_s. Negative damping also occurs at other locations of the center of rotation. In addition, negative damping may occur even at large M_∞ numbers, provided the amplitude of oscillations and the angle θ_s are such that the Mach number becomes sufficiently close to unity, at least on one side of the wedge.

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